A Study of Mixed Convection Heat Transfer in Different Structures

Aasheesh Bajpai*, Akhil Aravind*,

* Aerospace Engineering Department,
Indian Institute of Technology, Kanpur

Abstract- *If one wants to know whether convection heat transfer in a system can be considered to be in the forced, in the free, or in the mixed convection regime, one has to know about the boundaries and transition for every type of flow. In this paper, few studies have been presented to understand the procedure for analysis of finding the boundaries and transition for mixed convection heat transfer. Correlation and empirical relations for finding heat transfer (Nusselt number), friction factor for different orientation of different geometries which are found out by different authors from experimental studies or analytical/numerical studies. This paper shows this analysis of mixed convection correlation for some structures for different orientation.*

NOMENCLATURE

η	Non dimensionalised r co-ordinate
ρ_o, ρ_w	Reference density/density of the fluid at wall temperature (kg/m^3)
β	Co-efficient of thermal expansion of the working fluid (K^{-1})
θ	Non-dimensionalised temperature in vertical channel flow
ϕ	Non-dimensionalised temperature in semicircular duct flow
T_o	Reference temperature in vertical channel flow (K)
T_w	Wall temperature of the semicircular duct (K)
W_m	Mean non dimensional velocity in the axial direction
w_m	Mean velocity in the axial direction (m/s)
D	Hydraulic diameter (m)
L	Transverse dimension of the vertical channel & Length of the fin (m)
A	Axial pressure gradient (kg/s^2-m^2)
U_o	Reference velocity in vertical channel (m/s)
Gr	Grashof Number ($Gr = \frac{g\beta\Delta T D^3}{\nu^2}$)
Pr	Prandtl Number ($Pr = \frac{\nu}{\alpha}$)
Br	Brinkman Number ($Br = \frac{\mu U_O^2}{k\Delta T}$)
Ξ	Gr/Re
R_T	Temperature difference ratio ($R_T = \frac{T_2 - T_1}{\Delta T}$)
ε	ΞBr
Gz	Graetz number
Ra	Rayleigh Number (Gr.Pr)
Q''	Axial heat flux per unit length (kg/m-s^2)
R	Radius of the semicurcular duct (m)
Re	Reynolds number
Ri	Richardson number($\frac{Gr}{Re^2}$)
Nu	Nusselt number
Gz	Gratz number
C_f	Local friction factor
Ω	Buoyancy parameter(|Gr|/Re^2
μ	Viscosity

Subscripts:

b	Bulk means condition
w	Wall condition
f	Mean film condition

I. Introduction

If along with force flow situation if buoyant forces also develop due to density difference caused by temperature difference, the situation is termed as mixed convection heat transfer. The main problem is to identify the boundaries to differentiate between the different type of heat transfer regimes. The limit, up to which we can say that this is purely forced conduction and neglect the free or natural convection. Same for the limit of natural convection alone and neglecting the forced convection effect. In between those two extremes a situation may arise where the effect of both types are significant and equal. Outside these boundaries, the normal types of correlations may be expected to apply. In a mixed convection, both free convection and forced convection participate in the heat transfer process Free convection is negligible if, $Gr/Re_L \ll 1$ and forced convection is negligible if, $Gr/Re_L \gg 1$. Mixed convection regime is significant at the time of $Gr/Re_L \approx 1$. The force fluid flow direction can be a constraint, the relative buoyancy force and force flow direction. Three possible relative directions are possible buoyancy force and flow direction have the same direction (assistive or aiding flow), if it is opposite direction (opposing flow) and if the relative direction is perpendicular the this is termed as transverse flow. An example of upward flow is force flow in vertical plate in the same direction of buoyancy force (assistive flow) and opposite is opposing flow. Flow over a horizontal plate, cylinder and sphere are the examples of transverse flows. In assisting and transverse flows, buoyancy acts to enhance the rate of heat transfer associated with pure forced convection; in opposing flows, it acts to decrease this rate. The mixed convection can be correlated through a simple expression of the form,

$$Nu^n = Nu_f^n \pm Nu_N^n \quad (1)$$

For finding the correlation for Nusselt number for different geometries, some experimental correlations are used. The plus sign signifies the assistive or transverse and minus sign signifies the opposing flow. "The best correlation of data is often obtained for n =3, although values of 7/2 and 4 may be better suited for transverse flows involving horizontal plates and cylinders (or spheres), respectively. Equation (1) is the first approximation equation and for the general use. If some serious problem arises like in industry then one should go for experimentally verified correlations for different geometries (Incropera, 2011)[1].

One wants to know when heat transfer can be considered to be in the forced, in the free, or in the mixed convection regime. For knowing the boundaries between forced, mixed and natural convection first one has to understand about the pure free and force convection and Nusselt number empirical relationship for those. For the fluid flowing in the laminar region pure forced convection fluid with Pr=0.7 and constant heat flux.

$$Nu = 4.36 + \frac{\{0.036(Re)(Pr)(D/L)\}}{\{1 + 0.0011(Re)(Pr)(D/L)\}} \quad (2)$$

Meets with exact theoretical solution within $\pm 3\%$ for $10 \leq Re.\mathrm{Pr}(D/L) \leq 1000$.
Free convection laminar flow

$$Nu = C(\mathrm{Gr})^m(\mathrm{Pr})^n \quad (3)$$

Recommended value for C is 0.39.
For turbulent fully developed flow

$$Nu = 0.022(\mathrm{GRe})^{0.8}0.42 \quad (4)$$

Where Pr is not too far from unity.
For external (Laminar & turbulent flow) including flat and vertical cylinders

$$Nu = 0.11(\mathrm{GrPr})^{1/3} + (\mathrm{GrPr})^{1/0.1} \quad (5)$$

boundaries between free and forced convection can be identified if one assumes same order of magnitude for both type of flows.

$$Nu = 0.022(\mathrm{GRe})^{0.8}0.42 = 0.13(Gr_D Pr)^{1/3} \quad (6)$$

After manipulation equation obtained is

$$Re = 9.2(Gr_D)^{0.417}(Pr)^{-.108} \quad (7)$$

If on logarithmic limit 100% of this straight line is followed then it separates the boundaries [2].
If the boundaries of free and forced convection are known, then one can move forward for finding the Nusselt number and finally the heat transfer for different structures and different orientation and relative fluid flow for some specific application. Here, in this paper different types of structures have been shown. At the first, a general analysis scheme is first described using a simple case of flow through a vertical duct with constant duct wall temperatures. In the next section, a particular case of interest is dealt with in which viscous dissipation is neglected. This assumption becomes reasonable in a case like a heat transfer from a fin, in which the viscous heat dissipation is negligible when compared to the heat convected from the surface of the fin. Such a case of transfer through a semicircular finned surface for a prescribed value of heat flux and uniform wall temperature is examined. The second case which has been discussed in this paper is for mixed convection in a finned circular duct. The third case which has been discussed in this paper is for mixed convection through a sphere. In many situations analytical and empirical relations are fine to use but sometimes for experimental procedure is necessary to find it. Some of the studies presented in this paper are from analytical studies and some from the experiments.

II. Motivation And importance of the problem

Channel mixed convection is particularly important in the analysis of compact heat exchangers, electronic cooling systems, cooling cores of nuclear reactors, and chemical process technology. They are commonly studied for the boundary conditions of prescribed wall temperatures and prescribed wall heat fluxes. In a general case of mixed convection, the heat transfer rate, Nusselt number, temperature and velocity profiles are functions of buoyance forces, pressure gradient and viscous dissipation. In lot of devices used in industry which have force flow along with heated surface, uniform wall temperature closely approximates the industrial process. In electronic cooling application like circuit board array mixed heat transfer utilization is really useful. So, our main motivation for this paper is to establish a methodology from glean literature for analysis of mixed convection in different structures. It is imperative sometimes to know the correlations and empirical relations of mixed convection heat transfer, when it is creating some losses and also when one is trying to get advantage out of this.

III. METHODOLOGY USED

VERTICAL CHANNEL MIXED CONVECTION[3]-

The following assumptions are made in the study of mixed convection a vertical channel. The flow model is simplified to arrive at an analytical solution.

1. The Fluid is Newtonian with properties like thermal conductivity, thermal diffusivity, dynamic viscosity and thermal expansion coefficient assumed as constants.
2. The Boussinesq approximation ($\rho = \rho_o[1 - \beta(T - T_O)]$) holds good.
3. The channel is aligned along the X-axis and confined between the spatial y-coordinates -L/2 and +L/2. The wall at y=-L/2 is taken as the cool wall at Temperature (T_1) and the wall at y=+L/2 is as the hot wall at Temperature (T_2) in case of asymmetric heating. In the case of symmetric heating, both the wall temperatures are taken to be equal.
4. The flow is 2-D, steady, fully developed and laminar. The only non-zero component of velocity is along the X-axis. Variation of flow properties along the X-axis is zero since the flow is fully developed.

Governing Equation and Solution Methodology

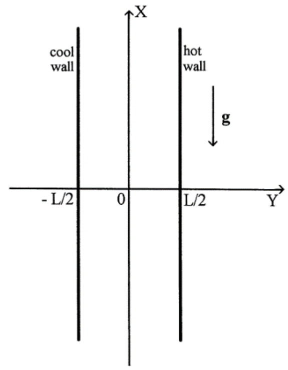

Figure 1: Schematic of the vertical channel

The mass conservation equation can be written as,

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho U)}{\partial X} + \frac{\partial(\rho V)}{\partial Y} = 0 \quad \Rightarrow \quad \frac{\partial U}{\partial X} = 0$$

The mass conservation equation implies that velocity is only a function of Y. Y-momentum conservation simplifies to a simple expression of zero pressure gradient in the Y-axis as the velocity component along the axis is zero. Therefore pressure becomes a function of X only. $\frac{\partial P}{\partial X}$ is the pressure gradient maintained across the ends of the pipe and can be assumed to be a constant(A). X - Momentum conservation equation can be written as,

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = 0 = \frac{-\partial P}{\rho_o \partial X} + \frac{\mu}{\rho_o}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) + \frac{\rho g}{\rho_o}$$
$$\Rightarrow \quad \frac{-A}{\rho_o} + \nu\left(\frac{d^2 U}{dY^2}\right) + \beta g(T - T_O) = 0 \tag{8}$$

T_o is the reference temperature and can be taken as the average of the wall temperatures. Similar to velocity, the temperature is only a function of Y. This can be verified by differentiating equation (8) with X to obtain an expression for (dT/dx). The energy conservation equation can be written as

$$\rho\frac{Dh}{Dt} = \frac{DP}{Dt} + k\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right) + \mu\left[2\left(\frac{\partial U}{\partial X}\right)^2 + 2\left(\frac{\partial V}{\partial Y}\right)^2 + \left(\frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y}\right)^2\right]$$
$$\Rightarrow \quad \alpha\left(\frac{\partial^2 T}{\partial Y^2}\right) + \frac{\nu}{C_P}\left(\frac{\partial U}{\partial Y}\right)^2 = 0 \tag{9}$$

Double differentiating equation(1) with respect to Y, yields an expression for $\frac{\partial^2 T}{\partial Y^2}$ which can be substituted in equation(2) to obtain a differential equation for U.

$$\frac{d^4U}{dY^4} = \frac{\beta g}{\alpha C_p}\left(\frac{dU}{dY}\right)^2 \tag{10}$$

The following parameters are defined for simplifying further analysis.

1. Hydraulic diameter, D = 2L
2. $\Delta T = T_2 - T_1$ for asymmetric heating and $\Delta T = \frac{\nu^2}{C_P D^2}$ for symmetric heating.
3. Reference velocity, $U_o = \frac{-48AD^2}{48\mu}$

The equation set and the boundary conditions can be non dimensionalised using the following dimensionless quantities,

$u = \frac{U}{U_O}$, $y = \frac{Y}{D}$, $\theta = \frac{T-T_O}{\Delta T}$, $Gr = \frac{g\beta\Delta T D^3}{\nu^2}$, $Re = \frac{U_O D}{\nu}$, $Pr = \frac{\nu}{\alpha}$, $Br = \frac{\mu U_O^2}{k\Delta T}$, $\Xi = \frac{Gr}{Re}$, $R_T = \frac{T_2 - T_1}{\Delta T}$

Equation (10) requires 4 boundary conditions for a particular solution. The no-slip boundary conditions can be invoked on the walls of the channel. Other boundary conditions can be obtained by evaluating equation (8) at the wall boundaries using the wall temperatures. The non dimensionalised form of the equation (10) along with its boundary conditions can be written as,

$$\frac{d^4u}{dy^4} = \Xi\mathrm{Br}\left(\frac{du}{dy}\right)^2 \tag{11}$$

$$u\left(-\frac{1}{4}\right) = u\left(+\frac{1}{4}\right) = 0 \tag{12}$$

$$\frac{d^2u}{dy^2}_{y=-1/4} = -48 + \frac{\Xi R_T}{2};\ \frac{d^2u}{dy^2}_{y=+\frac{1}{4}} = -48 - \frac{\Xi R_T}{2} \tag{13}$$

The equation sets and the boundary conditions prove that the velocity profile is a function of the ratio of Grashof number to Reynolds number (Ξ), Brinkman number (Br) and Temperature difference ratio (R_T).

The solution can be obtained by using the perturbation method in which the variable of interest is expressed in terms of the perturbation series of a dimensionless parameter. The dimensionless parameter is chosen based on the coefficient in the non dimensionalised differential equation. Therefore u(y) can be expressed in terms of $\varepsilon = Br\Xi$ as,

$$u(y) = u_o(y) + u_1(y)\varepsilon + u_2(y)\varepsilon^2 + u_3(y)\varepsilon^3 + \cdots u_n(y)\varepsilon^n \tag{14}$$

The expression for u(y) is substituted back in equation(4) and the coefficients of each power of ε are equated to zero to find the unknown functions ($u_o(y)$, $u_1(y)$,….). Once the velocity field is obtained, the temperature field can be found out by non-dimensionalising equation (8) and then simplifying for θ. Further simplification can be made by substituting for u(y) from equation (14)

$$\theta = -\frac{1}{\Xi}\left(48 + \frac{d^2u}{dy^2}\right) = 2R_T y - \frac{1}{\Xi}\sum_{n=1}^{\infty}\frac{d^2u_n(y)}{dy^2}\varepsilon^n \tag{15}$$

Nusselt number can be defined at boundaries as the gradient of non-dimensionalised temperature. Thus Nusselt number at the boundaries can be expressed in terms of u(y)

$$Nu_- = \left(\frac{d\theta}{dy}\right)_{y=-\frac{1}{4}} = 2R_T - \frac{1}{\Xi}\sum_{n=1}^{\infty}\left(\frac{d^3u_n(y)}{dy^3}\right)_{y=-\frac{1}{4}}\varepsilon^n \tag{16}$$

$$Nu_+ = \left(\frac{d\theta}{dy}\right)_{y=+\frac{1}{4}} = 2R_T - \frac{1}{\Xi}\sum_{n=1}^{\infty}\left(\frac{d^3u_n(y)}{dy^3}\right)_{y=+\frac{1}{4}}\varepsilon^n \tag{17}$$

MIXED CONVECTION IN A FINNED SEMI CIRCULAR DUCT[4]-

The following assumptions are made in the study of finned semi-circular ducts.

1. The flow is developed and laminar throughout the length of the duct. The velocity field is non-zero only along the axial direction. The flow is upward in such a way that buoyance force is in the direction of the main flow and the Boussinesq approximation ($\rho = \rho_w[1 - \beta(T - T_w)]$) holds good. ($\rho_w$ is the density of the fluid at wall temperature of T_w)

2. The wall of the duct is assumed to have a constant axial heat flux per unit length (Q''). The duct and the fins walls are assumed to be highly conductive so that the temperature profile is uniform across any cross-section.

3. The Fluid is Newtonian with properties like thermal conductivity, thermal diffusivity, dynamic viscosity and thermal expansion coefficient assumed constant.

4. Viscous dissipation and compression work terms are neglected in the energy equation.

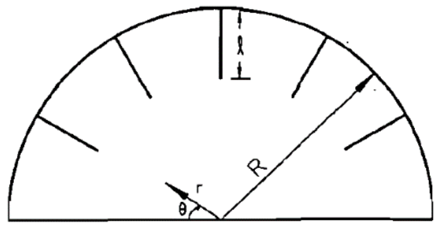

Figure 2: Cross section of the semicircular finned duct

5. The fins are evenly spaced around the inner circumference of the semicircular duct of radius R. The thickness of the fin is negligible and its length is l. The cross-section of the duct is shown in figure (2).

Governing Equations and the Solution Methodology

The mass conservation equation simplifies to $\frac{\partial(\rho w)}{\partial z} = 0$. The momentum conservation equation is only written along the axial direction. In the radial and tangential directions, the gradients are zero because the velocity components are zero.

$$-\rho_w[1-\beta(T-T_w)]g - \frac{dp}{dz} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right] = 0 \tag{18}$$

The energy equation can be simplified as,

$$\rho_w[1-\beta(T-T_w)]C_P w\frac{\partial T}{\partial z} = k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right] \tag{19}$$

The equation set can be non-dimensionalised using the following variables.

$$\eta = \frac{r}{R},\quad W = \frac{\mu w}{-R^2[(dp/dz)+\rho_w g]},\ \phi = \frac{\pi}{2}\left(\frac{T-T_W}{(Q''/k)}\right)W_m,\ Ra = Gr.pr = \ \rho_w g\beta R^4\left(\frac{dT_b}{dz}\right)/(\alpha\mu)$$

The governing equations reduce to,

$$\frac{1}{\eta}\frac{\partial}{\partial \eta}\left(\eta\frac{\partial W}{\partial \eta}\right) + \frac{1}{\eta^2}\frac{\partial^2 W}{\partial \theta^2} + (Ra.\phi) + 1 = 0 \tag{20}$$

$$\frac{1}{\eta}\frac{\partial}{\partial \eta}\left(\eta\frac{\partial \phi}{\partial \eta}\right) + \frac{1}{\eta^2}\frac{\partial^2 \phi}{\partial \theta^2} - W = 0 \tag{21}$$

The boundary conditions are W=0 and ϕ=0 on the walls and the fins. The axial temperature gradient can be estimated based on the heat flux input as,

$$\frac{\partial T}{\partial z} = \frac{Q''}{\rho C_P w_m} \tag{22}$$

The differential equation and its boundary conditions are discretised using the finite difference method and the equations are solved iteratively for W and ϕ until a converged solution is obtained. Once the velocity profile is known, its gradient at the boundaries can be used to evaluate the wall shear stress and friction coefficient. The gradient of the non-dimensionalized temperature can be used to evaluate the Nusselt number.

Mixed convection heat transfer about spheres[5]-

At large values of Reynolds and Grashof numbers this study has been carried which includes entire regime of mixed convection, with both aiding and opposing flow. In the analysis, the conservation equations of the boundary layer are transformed such that they can lend themselves to either Local non-similarity or finite-difference solutions. Very efficient and accurate finite-difference method has been employed for solving the system of transformed equation. Numerical solutions were carried out and results obtained for gases having a Prandtl number of 0.7, for both aiding and opposing flows. For the aiding flow, the solutions encompassed the range of buoyancy parameter Ω between 0 (pure forced convection) and ∞(pure free convection). The opposing flow solutions were for Ω between 0 and - 3.0.

Analysis-

Consider a sphere in undisturbed upcoming stream in opposing flow (gravity acting opposite to the forced flow direction). Radius of the sphere is R The surface of the sphere is maintained at a uniform temperature T_w. If, $T_w > T_\infty$ then the velocity field due to buoyancy is aiding in the force flow, if, $T_w < T_\infty$ in this case velocity vectors due to buoyancy oppose the forced flow. Let the coordinates be chosen such that x measures the distance along the surface of the sphere front. the lower stagnation point and t measures the distance normal to the surface. In case of opposing flow, direction of x changes.

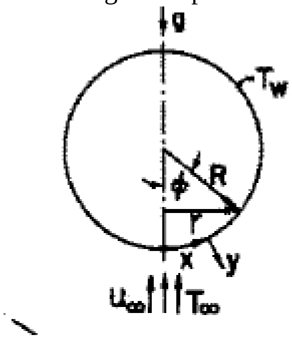

Figure 3 Sphere for analysis

The starting point of the analysis are following boundary layer equations-

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0$$

$$u\frac{\partial(u)}{\partial x} + v\frac{\partial(v)}{\partial y} = U\frac{\partial(u)}{\partial x} + v\frac{\partial^2(u)}{\partial y^2} + g\beta(T - T_\infty)\sin\frac{x}{R} = 0$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial(T)}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

Boundary conditions,

$$u = v = 0;\ T = Tw\ at\ y = 0$$
$$u \to u(x), T \to T_\infty\ as\ y \to \infty$$

The local free stream velocity U(x) in general has the expression-

$$\frac{U}{U_\infty} = A\left(\frac{x}{R}\right) + B\left(\frac{x}{R}\right)^3 + C\left(\frac{x}{R}\right)^5 + D\left(\frac{x}{R}\right)^7$$

Where A, B, C, D are constants.

$$\frac{U}{U_\infty} = \frac{3}{2}\sin\frac{x}{R} \quad (23)$$

with A = 3/2, B = -1/4, C = 1/80, D= -1/3380 etc. from sine series expansion.

The equation need transformation from (x,y) coordinates to (ξ, η) coordinate system. it is convenient to carry out the transformation of the conservation equations separately for forced-flow dominated and buoyancy-force dominated cases. The combination of the solutions from these two cases then encompasses the entire regime of mixed forced and free convection.

Forced flow dominated case-

$$C_f(Re)^{\frac{1}{2}} = 2^{\frac{1}{2}}\left[(\frac{U}{U_\infty})^2/(\xi)^{\frac{1}{2}}\right]\theta'(\xi), 0) \quad (24)$$

$$C_f(Gr)^{\frac{1}{4}} = \Omega^{\frac{1}{4}}C_f(Re)^{\frac{1}{2}} \quad (25)$$

$$NuRe^{-\frac{1}{2}} = -\left[(\frac{U}{U_\infty})/(2\xi)^{\frac{1}{2}}\right]\theta'(\xi), 0 \quad (26)$$

$$NuGr^{-\frac{1}{4}} = NuRe^{-\frac{1}{2}}/\Omega^{\frac{1}{4}} \quad (27)$$

For buoyancy dominated case-

When the buoyancy induced flow dominates over the forced convective flow, one examines the effects of the latter on the former. It is, therefore, appropriate to transform the conservation equations following the pattern that is used for pure free convection. In this connection, one employs the transformation variables.

$$X = \frac{x}{R}, Y = \frac{y}{r}\left|Gr^{1/4}\right|$$

along with the reduced stream function ***F(X, Y)*** and the dimensionless temperature θ(X, Y):

$$F(X,Y) = \frac{\psi(x,y)}{X|Gr|^{\frac{1}{4}}}, \emptyset(x,y) = \frac{T(x,y) - T_\infty}{T_w - T_\infty}$$

$$C_f(Re)^{\frac{1}{2}} = 2XF''(x,0)/\Omega^{\frac{3}{4}} \tag{28}$$

$$C_f(Gr)^{\frac{1}{4}} = C_f(Re)^{\frac{1}{2}}/\Omega^{\frac{1}{4}} \tag{29}$$

$$NuRe^{-\frac{1}{2}} = -\theta'(x,0)/\Omega^{\frac{1}{4}} \tag{30}$$

$$NuGr^{-\frac{1}{4}} = -\theta'(x,0) \tag{31}$$

In the numerical computations, which covered $0 \leq \Omega \leq \infty$(i.e.) for values ranging from pure forced convection to pure free convection), were used for$0 \leq \Omega \leq 10$ and equations for$1 \leq \Omega \leq \infty$It was verified that the two sets of equations yielded the same results.

IV. Results and discussion

Vertical channel mixed convection-
Asymmetric Heating (R_T = 1)

When $\frac{\partial P}{\partial X}$ is negative, the velocity field, Reynolds number, Ξ and ε are positive. The flow is upward. The temperature and velocity profiles are functions of both Ξ and ε. Even though ε is directly related to Ξ, ε signifies the viscous dissipation and Ξ signifies the effect of buoyance force on the flow. Figures(2) and (3) show the velocity and temperature profile when $\Xi = 100$ and $\Xi = 500$ respectively. The graphs are plotted for 3 different values of ε (0, 8 and 12). Increase in Ξ at a given ε signifies an increase in buoyance force for a given viscous dissipation. As ε increases, viscous dissipation increases, increasing the temperature at every point in the profile. As the temperature increases, the fluid gets lighter, increasing buoyance force which accelerates the flow velocity at every point in the profile. This acceleration is not uniform since the heating is asymmetric. The fluid closer to the hot wall (y=1/4) is accelerated more than the fluid at the cold wall (y=-1/4). As the buoyance effects become more significant, the non-uniformity in the velocity profile increases.

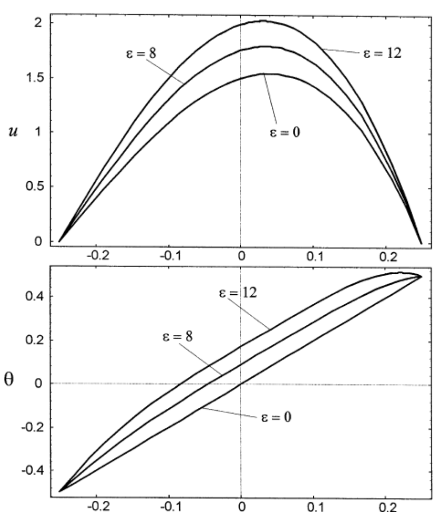

Figure 4: Velocity and Temperature profile for $\Xi = 100$

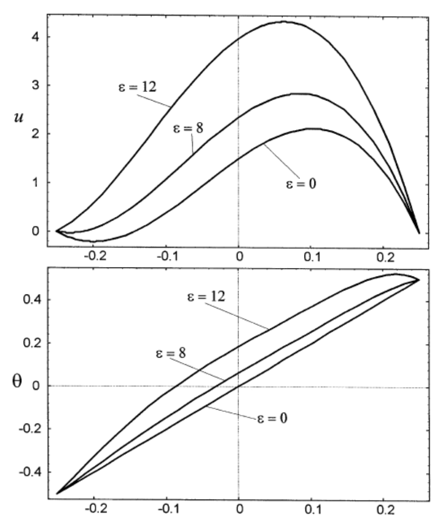

Figure 5: Velocity and Temperature profile for $\Xi = 500$

Beyond a certain value of Ξ, the velocity near the hot wall rises to an extent that the flow has to reverse near the cold wall to main a constant mass flow rate. This can be seen in figure(4) where Ξ has crossed the critical limit and the flow has reversed at the cold wall for ε=0(and 8). Viscosity has the effect of diffusing out the momentum and prevents flow reversal. As can be seen in figure(3), for ε=12, the flow reversal is not present at the cold wall.

Figure 4 and 5 show the velocity and temperature profiles for downward flow. For downward flow, Ξ and ε are negative due to the negative value of Uo. The variations are plotted for the same set of Ξ and ε as was plotted for upward flow. Since the flow is downwards, the increase in dissipation tends which increases the buoyance force tends to oppose the downward flow. This deceleration is more predominant near the hot wall due to its higher wall temperature. Beyond a certain negative value of Ξ, the flow reversal occurs at the hot wall as shown in figure(5). Comparison between the profiles of upward and downward flow shows that the upward flow is more significantly affected by viscous dissipation that downward flow.

Symmetric Heating (R_T = 0)

In the case of symmetric heating, the velocity and temperature profiles are symmetric. The effect of viscous dissipation is to increase the fluid temperature, which increases the buoyance force which accelerates the velocity in case of upward flow and decelerates in case of downward flow. Due to symmetric heating ($R_T = 0$), the effect on Ξ on the velocity profile vanishes as the Ξ always come coupled with R_T in the solution of perturbation series. Thus the velocity becomes on a function of ε. From equation(8), θ Ξ also becomes a function of ε only. The velocity and temperature profiles are shown in figure 6) and figure(7) respectively.

Even in the case of symmetric heating, beyond a certain Ξ, buoyance forces can reverse the flow at the walls. This can lead to the creation of local maxima/minima of velocity near the walls and a dip/rise in the velocity in the core region respectively to maintain a constant mass flow rate. This characteristic is examined in the next section of heat transfer in a finned semicircular duct.

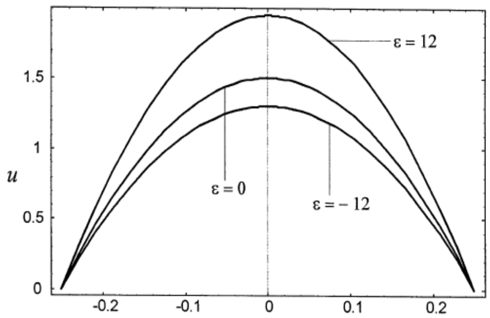

Figure 6: Symmetric heating velocity profile

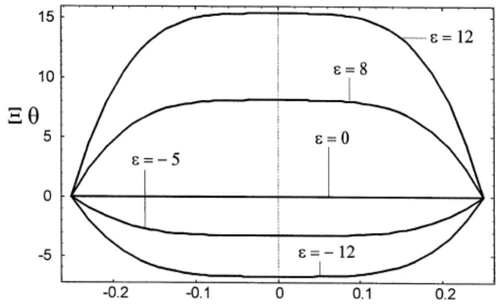

Figure 7: Symmetric heating temperature profile

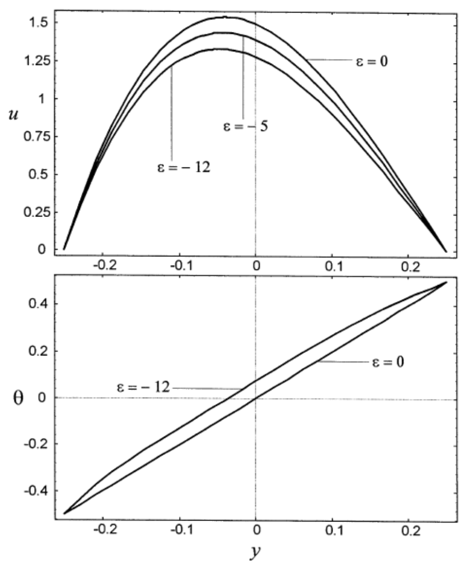

Figure 8: Velocity and Temperature profile for $\Xi = -100$

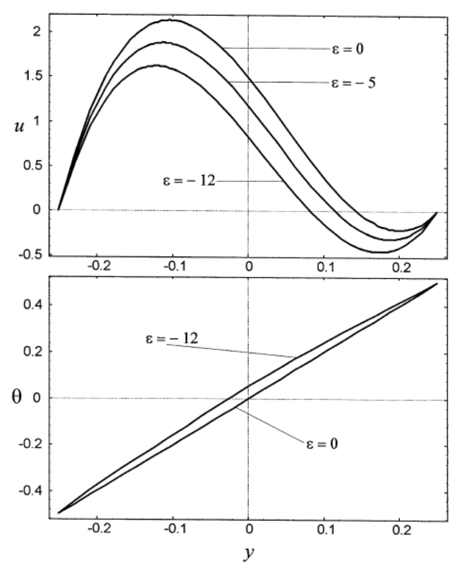

Figure 9: Velocity and Temperature profile for $\Xi = -500$

Mixed convection in a finned semi-circular duct-

The velocity and temperature profiles at different Rayleigh numbers for the for a 3 finned semi-circular duct with a fin length of 0.6R is shown in figure(10) and (11) respectively. The profiles are plotted for an angle of 67.5^{o} along the radial direction. 2 peaks and a valley is observed in the velocity profile for $Ra<10^4$ beyond which multiple peaks and valleys start appearing in the profile.

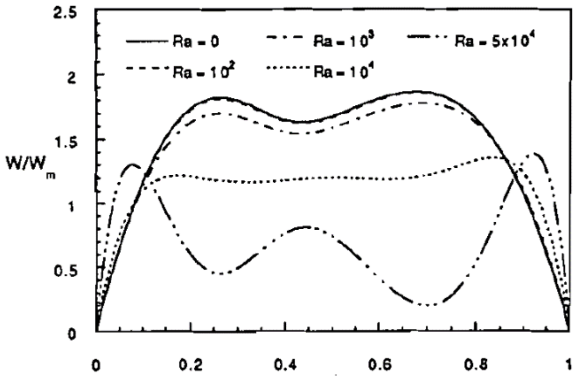

Figure 10: Velocity profile of semicircular duct

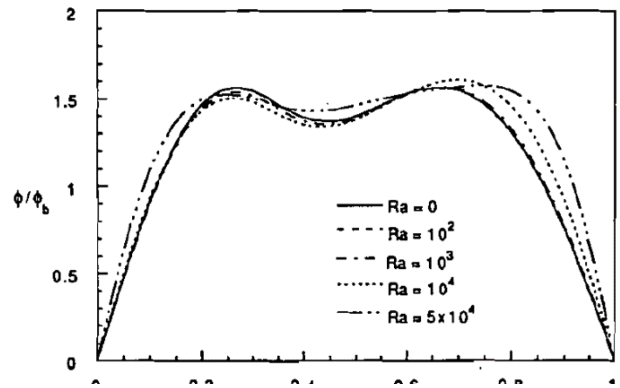

Figure 11: Temperature profile of the semicircular duct

The fluid near the walls is heated and accelerated by a buoyance force. This increase in fluid velocity near the walls has to be compensated by a reduction in the fluid velocity near the core to maintain the flow rate. This leads to a profile with local maxima near the walls and local minima near the core. As the Rayleigh number increases, the effects of buoyancy becomes more significant and thus the velocity profile becomes more distorted. The temperature profile also shows a similar dip near the core and a local maximum near the wall which are not much altered by a higher Rayleigh number.

As the Rayleigh number increases, both the temperature and the velocity profiles get steeper at the walls, increasing the friction coefficient and the Nusselt number. Thus the heat transfer rate is significantly increased by the effect of buoyance forces (natural convection). These changes are quite a visile for Ra>10^3. The variation of Nusselt number with Rayleigh number is better illustrated in figure(12) and figure(13). Nusselt number, in this case, is normalised with the Nusselt number for forced convection (Ra=0).

Figure(12) and (13) also illustrate the effects of fin dimensions and number of fins on the Nusselt number ratio $\left(\frac{Nu}{Nu_o}\right)$. The pattern does not seem to be monotonous. However, it can be inferred that, up to a critical limit on the number of fins and fin dimensions, the Nusselt

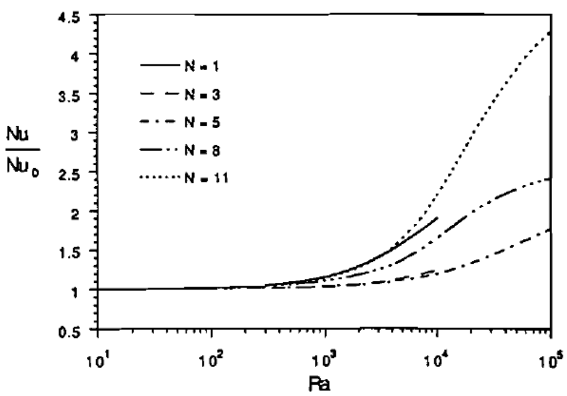

Figure 12: $\frac{Nu}{Nu_o}$ vs Ra for different number of fins

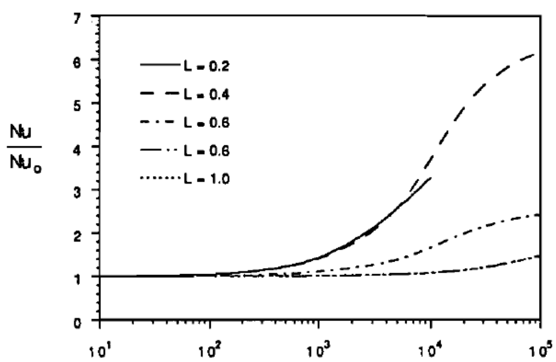

Figure 13: $\frac{Nu}{Nu_o}$ vs Ra for different fin lengths

number ratio is found to decrease with increase in the number of fins and fin dimension.

For mixed convection about sphere-

Numerical results were obtained for gases having a Prandtl number of 0.7. For aiding and opposite flow both local wall shear stress local surface heat transfer rate and temperature, velocity distribution has been included in the results. Since no experimental data is available,One of these is the potential flow solution given by equation (23) and measurements as given by equation (22) with A = 1.5, *B* = -0.4371, C = 0.1481 and D=-0.0423. Thus, the results to be presented will terminate at $\emptyset$= 90°. Figure (14) illustrates the angular distributions of the local wall shear stress for the two local free stream velocity distributions. It can be analysed from the figure (14) that C_f increases with increasing buoyancy force in the case of $Gr/Re^2>0$. This is because of the aiding flow situation. On the other hand, in the case of $Gr/Re^2>0$ opposite type of flow case C_f decreases with increasing buoyancy force. As a result flow separation occurs earlier as it reaches the stagnation point.

Figure (15) shows angular distribution of Nusselt number at Pr=0.7. value of Nusselt number increases so that the local heat transfer rate as the angle increases in case of aiding flow situation. But opposite trend has been observed in case of opposite flow situation. The relative changes in the local Nusselt number Nu/Nu_0 where Nu_0 is Nusselt number at stagnation point. From figure (16) it is clear that for very low to somewhat moderate buoyancy forces the Nusselt number strongly depend on the angle which is due to domination of the forced convection. Figure (17) provides a better understanding of local heat transfer for combined free and force convection (aiding flow) by providing the effect of buoyancy on local Nusselt number at three angular position 0°, 60°, 90° respectively. The asymptotes at the stagnation point (i.e. $\emptyset = 0°$) for pure forced convection ($Gr/Re^2 = 0$) and pure free convection ($Gr/Re^2 = \infty$).

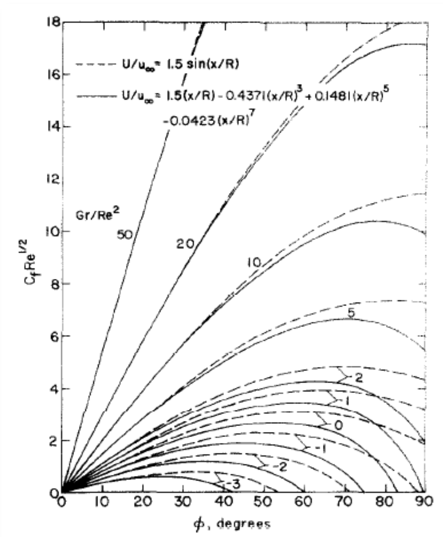

Figure 14 Angular distribution of local *friction factor*

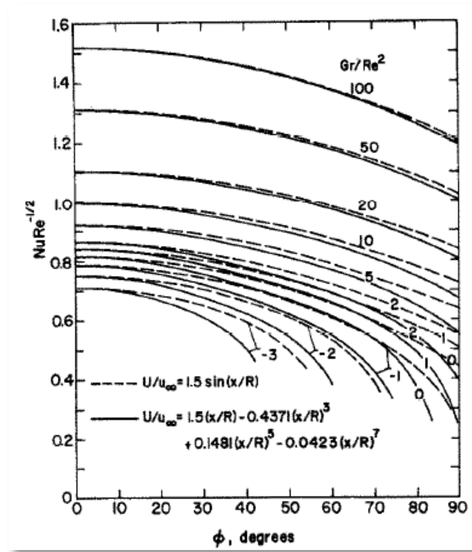

Figure 15 Angular distribution of Nusselt number for Pr=0.7

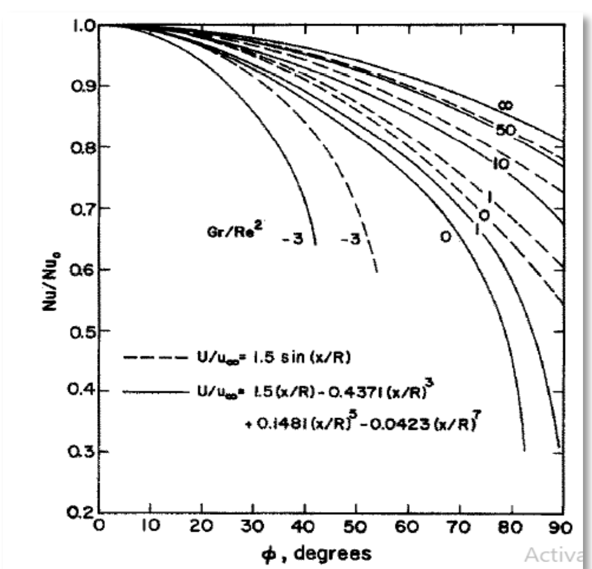

Figure 16 Relative angular dependence of Nusselt number for Pr=0.7

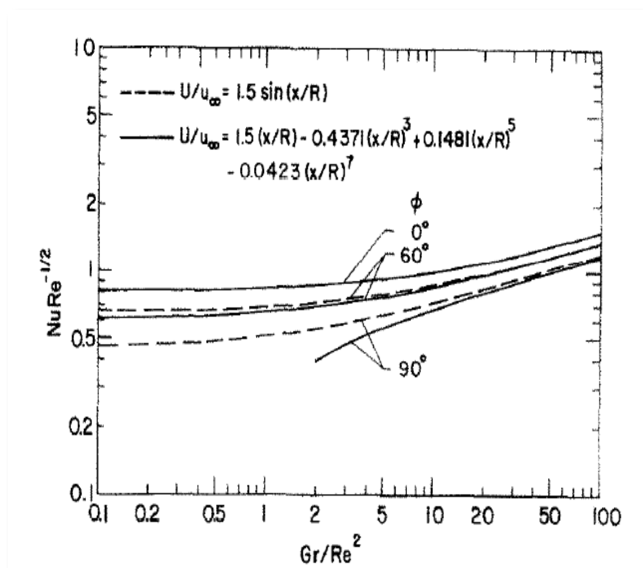

Figure 17 Heat transfer results at representative angular positions for Pr=0.7

$$NuRe^{-\frac{1}{2}} = 0.8149, NuRe^{-\frac{1}{2}} = 0.4576\Omega^{\frac{1}{4}}$$

It should be noted that the curve for the case of ($\emptyset = 90°$) with local free stream velocity distribution from measurements starts from $Gr/Re^2 = 2$. This is because the flow has already separated at ($\emptyset < 90°$) for $Gr/Re^2 < 2$.

V. Conclusion

The results show that there is a considerable enhancement in heat transfer rate due to the effect of buoyance forces, especially at high Rayleigh number (above 10^3). Thus neglecting the effect of body forces in the estimation of heat transfer rate in forced convection for high Rayleigh numbers leads to inaccurate results. In the mixed convection analysis, the temperature-velocity profiles and the heat transfer rate is dependent on viscous dissipation, pressure gradient and buoyance forces. The effect of viscosity becomes quite significant for high viscosity fluids and high speed flows. However, under certain conditions (like that a fin heat transfer), its effect can be neglected because its magnitude is negligible when compared to the heat convected by the fin surface.

In the case of flow past sphere, local friction coefficient increases with increasing buoyancy force in the case of $Gr/Re^2>0$. This is because of the aiding flow situation. On the other hand, in the case of $Gr/Re^2>0$ opposite type of flow case C_f decreases with increasing buoyancy force. As a result, flow separation occurs earlier as it reaches the stagnation point. Value of Nusselt number increases (also the local heat transfer rate) as the angle increases in case of aiding flow situation and opposite trend has been observed in case of opposite flow situation. Gases having Pr=0.7 has significant buoyancy effect on pure convection are encountered for $Gr/Re > 1.67$ for aiding flow situation and $Gr/re < -1.33$ for opposing flow situation. For both aiding and opposing flow, the local wall shear and local Nusselt number results exhibit a strong dependence on the variation of free stream velocity for small to moderate buoyancy forces.

References

[1] F.P. Incropera, Fundamentals of heat and mass transfer (7th ed.), 2011. https://doi.org/10.1007/978-3-319-15793-1_19.

[2] C.K. Brown, W.H. Gauvln, Combined FreeRand . Forced Convection I . Heat Transfer in Aiding Flow Wh, (1965).

[3] A. Barletta, Laminar mixed convection with viscous dissipation in a vertical channel, Int. J. Heat Mass Transf. 30 (1997).

[4] P. Taylor, Numerical Heat Transfer , Part A : Applications : An International Journal of Computation and Methodology "Analysis of combined natural and forced convection in vertical semicircular ducts with radial internal fins", (2007) 37–41. https://doi.org/10.1080/10407789508913706.

[5] T.S. Chen, A. Mucoglu, Analysis of mixed forced and free convection about a sphere, Int. J. Heat Mass Transf. 20 (1977) 867–875. https://doi.org/10.1016/0017-9310(77)90116-8.

.